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Plant capacity notions in a non-parametric framework: a brief review and new graph or non-oriented plant capacities

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Abstract
Output-oriented plant capacity in a non-parametric framework is a concept that has been rather widely applied since about twenty-five years. Conversely, input-oriented plant capacity in this framework is a notion of more recent date. In this contribution, we unify the building blocks needed for determining both plant capacity measures and define new graph or non-oriented plant capacity concepts. We empirically illustrate the differences between these various plant capacity notions using a secondary data set. This shows the viability of these new definitions for the applied researcher.

Keywords Data envelopment analysis · Technology · Capacity utilization

JEL Classification D24

1 Introduction

The concept of plant capacity has been introduced in the economic literature by Johansen (1968). Färe et al. (1989a, c) provide an operational way to measure this concept using a non-parametric frontier framework focusing on a single output and multiple outputs, respectively.

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Plant capacity utilisation can then be determined from data on observed inputs and outputs by computing a pair of output-oriented efficiency measures relative to a general specification of a non-parametric frontier technology. This has led to a series of empirical applications mainly in fisheries (e.g., Felthoven 2002) and in the health care sector (for instance, Karagiannis 2015). There have also occurred some methodological refinements. One example is the inclusion of this plant capacity notion in a decomposition of the Malmquist productivity index (see De Borger and Kerstens 2000).

More recently, (Cesaroni et al. 2017) use the same non-parametric frontier framework to define a new input-oriented measure of plant capacity utilisation based on a couple of input-oriented efficiency measures. Furthermore, (Kerstens et al. 2019b) argue and illustrate empirically that the output-oriented plant capacity utilization may be unrealistic because the amounts of variable inputs needed to reach the maximum capacity outputs may not be available. This relates to the so-called attainability issue described in Johansen (1968). In response, Kerstens et al. (2019b) define a new attainable output-oriented plant capacity utilization that bounds the available variable inputs.

First, we want to offer a brief review of the above developments in defining different plant capacity notions. Second, we want to take a new methodological step and show how new graph or non-oriented plant capacity concepts naturally follow from rewriting the existing output- and input-oriented plant capacity utilisation notions. In particular, we make use of a variation on the generalized Farrell graph efficiency measure that goes back to Färe et al. (1985). It is also the first time these graph plant capacity notions are empirically applied. These new plant capacity concepts are more general than the existing ones and provide new tools for the applied researcher. In fact, it can be shown that these new graph or non-oriented plant capacity concepts have a profit-like interpretation, just like the traditional output- and input-oriented plant capacity notions are related to revenue maximization and cost minimization, respectively.

This paper is structured as follows. Section 2 provides some basic definitions related to the technology and its representation. Section 3 summarizes the existing output- and input-oriented plant capacity utilisation notions and reports the similarities in the building blocks needed for these plant capacity notions. In Sect. 4 we propose the new graph or non-oriented plant capacity notions based on some existing graph or non-oriented efficiency measures. We also establish some relations between these different plant capacity notions. Section 5 develops a simple numerical example to illustrate the existing and new plant capacity notions within the simplest possible setting. Section 6 offers an empirical application using a secondary data set. The final section concludes.

2 Technology: Basic Definitions

This section introduces some basic notation and defines the production technology. Given an N-dimensional input vector $x \in \mathbb{R}^N_+$ and an M-dimensional output vector $y \in \mathbb{R}^M_+$, for every observed production unit $k = 1, \ldots, K$ the production possibility set or production technology T is defined as follows: $T = \{(x, y) \mid x \text{ can produce at least y}\}$. Associated with technology T, the input set denotes all input vectors x capable of producing at least a given output vector y: $L(y) = \{x \mid (x, y) \in T\}$. Analogously, the output set associated with T denotes all output vectors y that can be produced from at most a given input vector x: $P(x) = \{y \mid (x, y) \in T\}$.

In this contribution, we assume that the production technology T satisfies some combination of the following standard assumptions:

(T.1) Possibility of inaction and no free lunch, i.e., $(0, 0) \in T$ and if $(0, y) \in T$, then $y = 0$.
(T.2) T is a closed subset of $\mathbb{R}^N_+ \times \mathbb{R}^M_+$.
(T.3) Strong input and output disposal, i.e., if $(x, y) \in T$ and $(x', y') \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, then $(x', -y') \geq (x, -y) \Rightarrow (x', y') \in T$.
(T.4) T is convex.

Briefly commenting on these traditional assumptions on the production technology, it is useful to recall the following (see, e.g., Hackman 2008 for details). Inaction is feasible, and there is no free lunch. Technology is closed. We assume strong or free disposability of inputs and outputs in that inputs can be wasted and outputs can be discarded at no opportunity costs. Finally, technology is convex. In our empirical analysis not all these axioms are simultaneously maintained.[1]

The radial input efficiency measure characterizes the input set $L(y)$ completely. It can be defined as follows:

$$DF_i(x, y) = \min\{\theta \mid \theta \geq 0, \theta x \in L(y)\} = \min\{\theta \mid \theta \geq 0, (\theta x, y) \in T\}. \quad (1)$$

This radial input efficiency measure has the main property that it is smaller than or equal to unity ($DF_i(x, y) \leq 1$), with efficient production on the boundary (isoquant) of $L(y)$ represented by unity. Furthermore, the radial input efficiency measure has a cost interpretation (see, e.g., Hackman 2008).

The radial output efficiency measure offers a complete characterization of the output set $P(x)$ and can be defined as follows:

$$DF_o(x, y) = \max\{\varphi \mid \varphi \geq 0, \varphi y \in P(x)\} = \max\{\varphi \mid \varphi \geq 0, (x, \varphi y) \in T\}. \quad (2)$$

Its main properties are that it is larger than or equal to unity ($DF_o(x, y) \geq 1$), with efficient production on the boundary (isoquant) of the output set $P(x)$ represented by unity. In addition, this radial output efficiency measure has a revenue interpretation (e.g., Hackman 2008).

In the short run, we can partition the input vector x into a fixed (x^f) and variable part (x^v). In particular, we denote $x = (x^f, x^v)$ with $x^f \in \mathbb{R}^{N_f}_+$ and $x^v \in \mathbb{R}^{N_v}_+$ such that $N = N_f + N_v$. For convenience, we assume that all producers have the same subvectors of fixed and variable inputs. Fixed inputs are impossible to adjust in the short run, while variable inputs are under complete control of management (see Färe et al. 1994, Ch. 10). Alternative definitions of input fixity can, e.g., be found in Rasmussen (2011, Ch. 12) who distinguishes between physical and financial restrictions.

Similar to Färe et al. (1989c), a short-run technology $T^f = \{(x^f, y) \in \mathbb{R}^{N_f}_+ \times \mathbb{R}^M_+ \mid$ there exist some x^v such that (x^f, x^v) can produce at least $y\}$ and the corresponding input set $L^f(y) = \{x^f \in \mathbb{R}^{N_f}_+ \mid (x^f, y) \in T^f\}$ and output set $P^f(x^f) = \{y \mid (x^f, y) \in T^f\}$ can be defined. Note that technology T^f is in fact obtained by a projection of technology $T \subseteq \mathbb{R}^{N+M}_+$ into the subspace $\mathbb{R}^{N_f+M}_+$ (i.e., by setting all variable inputs equal to zero). By analogy, the same applies to the input set $L^f(y)$ and the output set $P^f(x^f)$.

Denoting the radial output efficiency measure of the short-run output set $P^f(x^f)$ by $DF^f_o(x^f, y)$, this short-run output-oriented efficiency measure can be defined as follows:

$$DF^f_o(x^f, y) = \max\{\varphi \mid \varphi \geq 0, \varphi y \in P^f(x^f)\} = \max\{\varphi \mid \varphi \geq 0, (x^f, \varphi y) \in T^f\}. \quad (3)$$

[1] For example, note that the convex variable returns to scale technology need not satisfy inaction.

The sub-vector input efficiency measure reducing only the variable inputs is defined as follows:

$$\begin{aligned} DF_i^{SR}(x^f, x^v, y) &= \min\{\theta \mid \theta \geq 0, (x^f, \theta x^v) \in L(y)\} \\ &= \min\{\theta \mid \theta \geq 0, (x^f, \theta x^v, y) \in T\}. \end{aligned} \tag{4}$$

This specification is mathematically equivalent to the work of Banker and Morey (1986).

Next, we need the following particular definition: $L(0) = \{x \mid (x, 0) \in T\}$ is the input set with zero output level. This is the input set indicating the input levels where non-zero production is initiated.[2] The sub-vector input efficiency measure reducing variable inputs evaluated relative to this input set with a zero output level is as follows:

$$\begin{aligned} DF_i^{SR}(x^f, x^v, 0) &= \min\{\theta \mid \theta \geq 0, (x^f, \theta x^v) \in L(0)\} \\ &= \min\{\theta \mid \theta \geq 0, (x^f, \theta x^v, 0) \in T\}. \end{aligned} \tag{5}$$

This sub-vector efficiency measure is defined with respect to the input set with zero output level where production is initiated.[3]

For the applications in Sects. 5 and 6 respectively, we assume a convex non-parametric frontier technology under the flexible or variable returns to scale assumption (VRS). Given data on K observations ($k = 1, \ldots, K$) consisting of a vector of inputs and outputs $(x_k, y_k) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$, this technology can algebraically be represented by

$$T^{VRS} = \left\{ (x, y) \mid x \geq \sum_{k=1}^{K} z_k x_k, \; y \leq \sum_{k=1}^{K} z_k y_k, \; \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \geq 0 \right\}. \tag{6}$$

The activity vector z of real numbers summing to unity represents the convexity axiom. The convex technology satisfies axioms (T.1) (except inaction) to (T.4).

Commonly, it is assumed that the input and output data satisfy a series of conditions (Färe et al. 1994, pp. 44–45): (i) each producer employs non-negative amounts of each input to produce non-negative amounts of each output; (ii) there is an aggregate production of positive amounts of every output as well as an aggregate utilization of positive amounts of every input; and (iii) each producer employs a positive amount of at least one input to produce a positive amount of at least one output.

3 Plant capacity concepts

3.1 Plant capacity concepts: a brief review of basic definitions

Recall the informal definition of plant capacity by Johansen (1968, p. 362) as "the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." This clearly output-oriented plant capacity notion has been admirably made operational by Färe et al. (1989a, c)

[2] Note that $L(0)$ can be equivalently defined by $L(y_{min}) = \{x \mid (x, y_{min}) \in T\}$, whereby $y_{min} = \min_{k=1,\ldots,K} y_k$. Thus, the minimum is taken in a component-wise manner for every output y over all observations K.

[3] This sub-vector input efficiency measure $DF_i^{SR}(x^f, x^v, 0)$ can be equivalently formulated as $DF_i^{SR}(x^f, x^v, y_{min})$, where $y_{min} = \min_{k=1,\ldots,K} y_k$ whereby the minimum is taken in a component-wise manner for every output over all observations.

using a pair of output-oriented efficiency measures. We now recall the definition of this output-oriented plant capacity utilization.

Definition 3.1 The output-oriented plant capacity utilization PCU_o is defined as follows:

$$PCU_o(x, x^f, y) = \frac{DF_o(x, y)}{DF_o^f(x^f, y)},$$

where $DF_o(x, y)$ and $DF_o^f(x^f, y)$ are output efficiency measures including, respectively excluding, the variable inputs as defined before in (2) and (3).

Since $1 \leq DF_o(x, y) \leq DF_o^f(x^f, y)$, notice that $0 < PCU_o(x, x^f, y) \leq 1$. Thus, output-oriented plant capacity utilization has an upper limit of unity. Following the terminology introduced by Färe et al. (1989a), one can distinguish between a so-called biased plant capacity measure $DF_o^f(x^f, y)$ and an unbiased plant capacity measure $PCU_o(x, x^f, y)$ depending on whether the measure ignores inefficiency or adjusts for the eventual existence of inefficiency. Taking the ratio of efficiency measures eliminates any existing inefficiency and yields in this sense a cleaned concept of output-oriented plant capacity. This output-oriented plant capacity utilisation compares the maximum amount of outputs with given inputs to the maximum amount of outputs in the sample with potentially unlimited amounts of variable inputs. It answers the question how the current amount of efficient outputs relates to the maximal possible amounts of efficient outputs.

More recently Yang and Fukuyama (2018) and Yang et al. (2019) provide an equivalent definition of $PCU_o(x, x^f, y)$ using an output-oriented directional distance function: it is well-known that the above radial output-oriented efficiency measures can be related to similar output-oriented directional distance functions (see Färe and Grosskopf 2000 for details). The originality of their approach is that these authors also distinguish between good and bad outputs: the good outputs are expanded, while the bad outputs are reduced.

Recently, Kerstens et al. (2019b) have argued and empirically illustrated that the output-oriented plant capacity utilization $PCU_o(x, x^f, y)$ may be unrealistic in that the amounts of variable inputs needed to reach the maximum capacity outputs may simply be unavailable at either the firm or the industry level. This is linked to what (Johansen 1968) called the attainability issue. Hence, Kerstens et al. (2019b) define a new attainable output-oriented plant capacity utilization at the firm level as follows:

Definition 3.2 An attainable output-oriented plant capacity utilization $APCU_o$ at level $\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$APCU_o(x, x^f, y, \bar{\lambda}) = \frac{DF_o(x, y)}{ADF_o^f(x^f, y, \bar{\lambda})},$$

where the attainable output-oriented efficiency measure ADF_o^f at a certain level $\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$\begin{aligned} ADF_o^f(x^f, y, \bar{\lambda}) &= \max\{\varphi \mid \varphi \geq 0, 0 \leq \theta \leq \bar{\lambda}, \varphi y \in P(x^f, \theta x^v)\} \\ &= \max\{\varphi \mid \varphi \geq 0, 0 \leq \theta \leq \bar{\lambda}, (x^f, \theta x^v, \varphi y) \in T\}. \end{aligned} \quad (7)$$

Again, for $\bar{\lambda} \geq 1$, since $1 \leq DF_o(x, y) \leq ADF_o^f(x^f, y, \bar{\lambda})$, notice that $0 < APCU_o(x, x^f, y, \bar{\lambda}) \leq 1$. Also, for $\bar{\lambda} < 1$, since $1 \leq ADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o(x, y)$, notice that $1 \leq APCU_o(x, x^f, y, \bar{\lambda})$. Kerstens et al. (2019b) pragmatically experiment with values of $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$. Furthermore, these authors note that if expert opinion cannot determine a plausible value, then it may be better to opt for the next input-oriented plant capacity measure that does not suffer from the attainability issue.

This attainable output-oriented plant capacity utilisation compares the maximum amount of outputs with given inputs to the maximum amount of outputs in the sample with amounts of variable inputs scaled by $\bar{\lambda}$. It answers the question how the current amount of efficient outputs relates to the maximal possible amounts of efficient outputs as determined by the scalar $\bar{\lambda}$.

Cesaroni et al. (2017) define a new input-oriented plant capacity measure using a pair of input-oriented efficiency measures.

Definition 3.3 The input-oriented plant capacity utilization (PCU_i) is defined as follows:

$$PCU_i(x, x^f, y) = \frac{DF_i^{SR}(x^f, x^v, y)}{DF_i^{SR}(x^f, x^v, 0)},$$

where $DF_i^{SR}(x^f, x^v, y)$ and $DF_i^{SR}(x^f, x^v, 0)$ are the sub-vector input efficiency measures defined in (4) and (5), respectively.

Since $0 < DF_i^{SR}(x^f, x^v, 0) \leq DF_i^{SR}(x^f, x^v, y)$, notice that $PCU_i(x, x^f, y) \geq 1$. Thus, input-oriented plant capacity utilization has a lower limit of unity. Similar to the previous case, one can distinguish between a so-called biased plant capacity measure $DF_i^{SR}(x^f, x^v, 0)$ and an unbiased plant capacity measure $PCU_i(x, x^f, y)$, the latter being cleaned of any prevailing inefficiency. This input-oriented plant capacity utilisation compares the minimum amount of variable inputs for given amounts of outputs with the minimum amount of variable inputs with output levels where production is initiated. It answers the question how the amount of variable inputs compatible with the initialisation of production must be scaled up to produce the current amount of outputs.

We end this brief review by pointing out two further sources of information. First, graphical illustrations of all plant capacity concepts are provided in online supplementary material A. Second, full details on the linear programming models to solve for all of these plant capacity concepts is found in online supplementary material B.1.

3.2 Plant capacity concepts: a digression

The purpose is now to develop a bottom-up approach whereby we start from the structure of the above existing output- and input-oriented plant capacity utilization concepts to develop a framework for new graph plant capacity utilization concepts. The following Proposition 3.1 presents a first new result. It shows that the building blocks needed for calculating the amount of $PCU_o(x, x^f, y)$ and $PCU_i(x, x^f, y)$ can be expressed in similar models (i.e., maximization for output-orientation and minimization for input-orientation) using the same constraints but with different objective functions and different bounds on the decision variables.

Proposition 3.1 (i) *The short-run output-oriented radial technical efficiency measure $DF_o^f(x^f, y)$ from model (3) is equivalently solved as follows:*

$$DF_o^f(x^f, y) = \max\{\varphi \mid \theta \geq 0, \varphi \geq 0, (x^f, \theta x^v, \varphi y) \in T\}, \tag{8}$$

whereby $\theta \geq 0$ allows to expand the observed variable inputs.

(ii) *The short-run input-oriented efficiency measure reducing variable inputs evaluated relative to the input set with a zero output level ($DF_i^{SR}(x^f, x^v, 0)$) from model (5) is equivalently solved as follows:*

$$DF_i^{SR}(x^f, x^v, 0) = \min\{\theta \mid \theta \geq 0, \varphi \geq 0, (x^f, \theta x^v, \varphi y) \in T\}, \tag{9}$$

whereby $\varphi \geq 0$ allows for an adjustment of the observed outputs.

iii) *The output-oriented technical efficiency measure $DF_o(x, y)$ from model (2) is equivalently solved as follows:*

$$DF_o(x, y) = \max\{\varphi \mid \theta \leq 1, \varphi \geq 1, (x^f, \theta x^v, \varphi y) \in T\}, \tag{10}$$

whereby $\theta \leq 1$ allows to contract the observed variable inputs.

iv) *The input efficiency measure reducing only the variable inputs ($DF_i^{SR}(x^f, x^v, y)$) from model (4) is equivalently solved as follows:*

$$DF_i^{SR}(x^f, x^v, y) = \min\{\theta \mid \theta \leq 1, \varphi \geq 1, (x^f, \theta x^v, \varphi y) \in T\}, \tag{11}$$

whereby $\varphi \geq 1$ allows for an adjustment of the observed outputs.

Proof See online supplementary material C. □

We can make the following remarks regarding this first new result. Although these remarks are more general by nature, it might be useful checking out the corresponding models in online supplementary material B assuming the convex non-parametric technology T^{VRS} [see (6)]. First, it is important to understand that in these new formulations, expressions (8)–(11) all use the same constraints (i.e., $(x^f, \theta x^v, \varphi y) \in T$). In the cases of output-orientation, maximization is needed while input-orientation requires minimization. Also notice the difference in the objective functions (i.e., φ in the case of output-orientation and θ for input-orientation). In particular, model (8) aims to maximize the outputs by releasing the variable inputs, while model (9) aims to minimize the variable inputs by releasing the outputs. The same result holds true for models (10) and (11).

Second, notice that models (8) and (10) are identical except for the bounds applied to the decision variables θ and φ. For $DF_o^f(x^f, y)$, we have $\theta \leq 1$ and $\varphi \geq 1$ that prevent to increase the inputs and decrease the output components, while for $DF_o(x, y)$, we have $\theta \geq 0$ and $\varphi \geq 0$. The same result holds true for models (9) and (11).

Third, note that in the new formulation of $DF_i^{SR}(x^f, x^v, 0)$ [i.e., model (9)], the right-hand side of the output constraints is not zero. In fact, this model aims to obtain the minimum amount of variable inputs such that the outputs are not restricted.

Since the input- and output-oriented plant capacity utilisation concepts share the same structure, we are now in a position to extend these notions to the full space of inputs and outputs by defining proper graph plant capacity utilization concepts.

4 Graph efficiency measurement and plant capacity utilisation

4.1 New developments

Methodological research on efficiency (or inefficiency) measurement has early on focused on measurement in the full space of inputs and outputs, which has been referred to as "graph efficiency" measurement in the seminal book by Färe et al. (1985). An extensive survey of such graph or non-oriented efficiency measures is provided by Russell and Schworm (2011).[4]

[4] A survey of similar input-oriented efficiency measures can be found in the earlier article of Russell and Schworm (2009).

Färe et al. (1985, pp. 110–111) define the hyperbolic efficiency index by

$$E_H(x, y) = \max\{\theta \mid (\theta^{-1}x, \theta y) \in T\}, \tag{12}$$

which can be considered as the first formulation of an efficiency index in the full input and output space. This index contracts inputs and expands outputs along a (particular) hyperbolic path to the frontier and maps into the $[1, \infty)$ interval. This hyperbolic graph efficiency measure (12) extends the analysis of the radial input- and output-oriented efficiency measures by allowing for the adjustment of both inputs and outputs in the measurement of efficiency. However, this hyperbolic graph efficiency measure is rather restrictive since it constrains the search for more efficient production plans to a hyperbolic path along which all inputs are reduced and all outputs are increased in the same proportion. Färe et al. (2002) show that the hyperbolic graph efficiency measure can be given a ratio-based return to the dollar interpretation.[5]

Under constant returns to scale (CRS), Färe et al. (2002) show that this hyperbolic efficiency index is equal to both $DF_i(x, y)^{\frac{1}{2}}$ and $DF_o(x, y)^{-\frac{1}{2}}$ (i.e., the conventional output- and input-oriented distance functions that can be solved using standard linear programming (LP) techniques). But, under VRS, the hyperbolic efficiency index may not be obtained by solving an LP-problem. To linearise this problem, Färe et al. (1989b) introduce a linear approximation of the non-linear set of constraints. However, Zofío and Lovell (2001) and Zofío and Prieto (2001) show that this approximation is only acceptable close to the efficient frontier. Hence, when a unit becomes more inefficient the approximation worsens. To resolve the linearisation problem of the hyperbolic distance function under VRS, Färe et al. (2016) propose an LP-based computational algorithm for estimating the exact value of the hyperbolic graph efficiency measure by connecting it to the directional distance function proposed by Chambers et al. (1998). We refer to Sect. 4.2 for more information concerning this directional distance function. Further computational results for the hyperbolic efficiency index are developed in the recent contributions of Halická and Trnovská (2019) and Hasannasab et al. (2019).

Starting from Färe et al. (1985, p. 126) one can define the generalized Farrell graph efficiency measure as follows:

$$E_{FGL}(x, y) = \max\left\{\frac{\varphi + \theta}{2} \mid \theta \geq 0, \varphi \geq 0, (\theta^{-1}x, \varphi y) \in T\right\}. \tag{13}$$

Proposition 4.1 *Even for strongly efficient units, $E_{FGL}(x, y)$ can be strictly greater than* 1*. However, if the constraints $\theta \geq 1$ and $\varphi \geq 1$ are added to model* (13)*, then for all strongly efficient units $E_{FGL}(x, y) = 1$.*

Proof See online supplementary material C. □

Based on the Proposition 4.1, we modify and relabel model (13) as follows:

$$E_{FGL}(x, y) = \max\left\{\frac{\varphi + \theta}{2} \mid \theta \geq 1, \varphi \geq 1, (\theta^{-1}x, \varphi y) \in T\right\}. \tag{14}$$

Note that $E_{FGL}(x, y)$ is larger than or equal to unity. This generalization of the hyperbolic efficiency measure permits the proportional reduction in all inputs to differ from the proportional increase in all outputs when searching for a more efficient production plan. Färe et al.

[5] See Halická and Trnovská (2019) for more historical details and for new duality results.

(1985, p. 154) also define the non-radial Russell graph measure of technical efficiency by decreasing inputs and increasing outputs in a non-radial way. In this formulation, they do consider the constraints $\theta_i \geq 1$ and $\varphi_r \geq 1$ similar to $\theta \geq 1$ and $\varphi \geq 1$ in (14). Halická and Trnovská (2018, pp. 391 and 395) show that model (14) in both its radial and non-radial variations can be interpreted as a shadow optimal profit.

A practical difficulty with this measure (14) is that it must be computed from a non-linear programming problem whose solution is not easily obtained. Therefore, we propose the alternative graph efficiency measure

$$E_G(x, y) = \max \left\{ \frac{\varphi}{\theta} \mid \theta \leq 1, \varphi \geq 1, (\theta x, \varphi y) \in T \right\}, \tag{15}$$

which, although closely related, avoids this difficulty. Instead of combining input and output radial measures in an additive way [see expression (14)], we now define the graph efficiency measure as the ratio between these input and output component measures. This model (15) has a profit interpretation (see online supplementary material B for details).

Contrary to $E_{FGL}(x, y)$, $E_G(x, y)$ can more easily be computed since it requires solving an ordinary linear fractional programming problem which can be achieved using linear programming by applying the Charnes and Cooper (1962) transformation (see online supplementary material B for details)[6]

Using the same structure, the sub-vector graph efficiency measure $E_G^f(x^f, x^v, y)$ is defined by

$$E_G^f(x^f, x^v, y) = \max \left\{ \frac{\varphi}{\theta} \mid \theta \leq 1, \varphi \geq 1, (x^f, \theta x^v, \varphi y) \in T \right\}. \tag{16}$$

It simultaneously reduces all variable inputs and expands all outputs.

The sub-vector graph efficiency measure $E_G^{SR}(x^f, x^v, y)$ defined by

$$E_G^{SR}(x^f, x^v, y) = \max \left\{ \frac{\varphi}{\theta} \mid \theta \geq 0, \varphi \geq 0, (x^f, \theta x^v, \varphi y) \in T \right\}, \tag{17}$$

gives complete freedom to adjust both the variable inputs as well as the outputs. Notice that models (16) and (17) are identical except for the bounds on the decision variables φ and θ.

Both efficiency measures $E_G^f(x^f, x^v, y)$ and $E_G^{SR}(x^f, x^v, y)$ aim to maximize the ratio of changes in outputs over changes in variable inputs. But, the main difference between these efficiency measures is as follows. In the efficiency measure $E_G^f(x^f, x^v, y)$, the output components increase and the variable inputs decrease, while in the short-run efficiency measure $E_G^{SR}(x^f, x^v, y)$ both variable inputs and output components are allowed to adjust in a flexible way.

Note that if the variable φ is ignored in the objective function of models (16) and (17) determining $E_G^f(x^f, x^v, y)$ and $E_G^{SR}(x^f, x^v, y)$, we then obtain the reciprocal of the efficiency measures $DF_i^{SR}(x^f, x^v, y)$ and $DF_i^{SR}(x^f, x^v, 0)$, determined by models (11) and (9), respectively. Similarly, if the variable θ is ignored in the objective function of models (16) and (17), we then obtain the efficiency measures $DF_o(x, y)$ and $DF_o^f(x^f, x^v, y)$, determined by models (10) and (8), respectively.

We now introduce different graph plant capacity notions using the above defined input-oriented, output-oriented and graph efficiency measures.

[6] Note that Pastor et al. (1999) proceed in a similar way to transform the non-linear part of the non-radial Russell graph measure proposed by Färe et al. (1985, p. 154). Thereafter, Tone (2001) extends this proposal into the so-called slack-based measure (SBM). Sueyoshi and Sekitani (2007, Theorem 1) prove that a nonradial version of $E_G(x, y)$ is less than or equal to the nonradial version of $E_{FGL}(x, y)$.

Definition 4.1 The graph non-oriented plant capacity utilization $GPCU$ is defined as follows:

$$GPCU(x, x^f, y) = \frac{E_G^f(x^f, x^v, y)}{E_G^{SR}(x^f, x^v, y)}. \tag{18}$$

Note that $GPCU(x, x^f, y) \leq 1$ since $0 < E_G^f(x^f, x^v, y) \leq E_G^{SR}(x^f, x^v, y)$. Thus, graph non-oriented plant capacity utilization has an upper limit of unity, but no lower limit.

It is important to realize that the denominators of the output-, input-, and graph-oriented plant capacity utilization measures in Definitions 3.1, 3.3 and 4.1 have different returns to scale characteristics. First, the output-oriented efficiency measure $DF_o^f(x^f, y)$ adjusts the variable inputs to obtain the maximum amount of outputs. The target point obtained from it has decreasing return to scale (DRS). Second, the input-oriented efficiency measure $DF_i^{SR}(x^f, x^v, 0)$ adjusts the outputs to obtain the minimum amount of variable inputs. Therefore, the obtained target points by solving this model $DF_i^{SR}(x^f, x^v, 0)$ has increasing return to scale (IRS). Finally, the new graph efficiency measure $E_G^{SR}(x^f, x^v, y)$ adjusts both variable inputs and outputs to obtain the maximum ratio of changes in outputs over changes in variable inputs. The target point obtained from it has constant returns to scale (CRS) or the most productive scale size (mpss).[7] As can be seen in Fig. 1 in the numerical example in Sect. 5 below, the target points obtained by solving the models for $DF_o^f(x^f, y)$, $DF_i^{SR}(x^f, x^v, 0)$ and $E_G^{SR}(x^f, x^v, y)$ are located on the DRS, IRS and CRS (or mpss) part of the efficient frontier.

This graph non-oriented plant capacity utilization compares the maximum amount of a ratio of outputs over variable inputs by expanding the outputs and contracting the variable inputs (i.e., by moving into the dominating region of the evaluated unit) to the maximum amount of a ratio of outputs over variable inputs in the sample with potentially unlimited amounts of both outputs and variable inputs, whence it is smaller than unity. It answers the question of how the current amount of efficient ratio of outputs over variable inputs relates to the maximum possible amount of the efficient ratio of outputs over variable inputs (see online supplementary material D for more details).

We have by definition no limitations on the available amounts of variable inputs for the original output-oriented plant capacity utilisation $PCU_o(x, x^f, y)$. However, in some empirical settings this is not realistic and we have to limit the amount of variable inputs available at either the firm or the industry level (see Kerstens et al. 2019b for details). One way to limit the amounts of variable inputs is as follows. It may be reasonable that the amount of increase in the variable inputs is proportional to the increase in the amount of outputs. Hence, we can define an output-oriented plant capacity utilisation in graph space by considering the changes of inputs.

[7] Banker (1984, p. 37) states: "the mpss for a given input and output mix is the scale size at which the outputs produced 'per unit' of the inputs is maximized. Thus, a production possibility $(x, y) \in T$ represents a mpss if and only if for all production possibilities $(\beta X, \alpha Y) \in T$ we have $\frac{\alpha}{\beta} \leq 1$." The model introduced by Banker (1984) for determining mpss is as follows:

$$\max \left\{ \frac{\alpha}{\beta} | \beta \geq 0, \alpha \geq 0, (\beta X, \alpha Y) \in T \right\}. \tag{19}$$

Note that by decomposing inputs into their fixed and variable components, the above mpss model (19) can be written as model (17): i.e., $E_G^{SR}(x^f, x^v, y)$.

This graph output-oriented plant capacity utilization can now be defined as follows:

Definition 4.2 The graph output-oriented plant capacity utilization $GPCU_o$ is defined as follows:

$$GPCU_o(x, x^f, y) = \frac{DF_o(x, y)}{GDF_o^{SR}(x^f, x^v, y)}, \tag{20}$$

where $GDF_o^{SR}(x^f, x^v, y)$ is the optimal value of φ in model (17).

Note that $GPCU_o(x, x^f, y) \gtreqless 1$ since $DF_o(x, y) \gtreqless GDF_o^{SR}(x^f, x^v, y)$. This lack of bound may create some difficulties in interpretation. If researchers prefer boundedness, then we recommend to use the traditional or attainable output-oriented plant capacity measure (Definition 3.1 or Definition 3.2). This graph output-oriented plant capacity utilization $GPCU_o$ compares the maximum amount of outputs with given inputs to the maximum amount of outputs with input levels where production is an mpss. It answers the question how the amount of outputs compatible with the mpss production must be scaled down or up to be compatible with the current amount of inputs (see online supplementary material D for more details).

As already mentioned in Sect. 3, the original output-oriented plant capacity utilization PCU_o suffers from the attainability issue. Hence, Kerstens et al. (2019b) introduce $APCU_o$ (see Definition 3.2) by imposing bounds on the availability of its variable inputs in a general way. Definition 4.2 can also provide another approach to solve the attainability issue. In fact, if we ignore the variable θ in the objective function of model (17), then $GPCU_o(x, x^f, y)$ becomes $PCU_o(x, y)$. Actually, the variable θ in the objective function prevents an increase of the variable inputs and allows the variable inputs to increase only as far as the ratio of changes in outputs over changes of variable inputs is maximized. However, in the following proposition, we show that $GPCU_o$ is a special case of $APCU_o$.

Proposition 4.2 *Assume that θ^* is the optimal value of θ in model (17). Then, we have:*

i) $GDF_o^{SR}(x^f, x^v, y) = ADF_o^f(x^f, y, \theta^*)$;
ii) $GPCU_o(x, x^f, y) = APCU_o(x, x^f, y, \theta^*)$.

Proof See online supplementary material C. □

Proposition 4.2 shows that if we use the attainable level $\bar{\lambda} = \theta^*$, then the graph output-oriented and the attainable output-oriented plant capacity utilization concepts offer the same results. Also, since the obtained target of $E_G^{SR}(x^f, x^v, y)$ is an mpss, then the obtained target from $ADF_o^f(x^f, y, \bar{\lambda})$ at a certain level $\bar{\lambda} = \theta^*$ is also an mpss. Note that $APCU_o(x, x^f, y, \bar{\lambda})$ depends on the attainable level $\bar{\lambda}$ that is determined by the decision maker. But, in some situations he or she may not be able to determine this attainability level. Therefore, the defined attainability in Definition 4.2 can offer a good choice to determine some reasonable bound on available variable inputs, because the attainable output-oriented efficiency measure $ADF_o^f(x^f, y, \bar{\lambda})$ at a certain level $\bar{\lambda} = \theta^*$ yields an mpss target.

In a similar way, the graph input-oriented plant capacity utilization can be defined as:

Definition 4.3 The graph input-oriented plant capacity utilization ($GPCU_i$) is defined as follows:

$$GPCU_i(x, x^f, y) = \frac{DF_i^{SR}(x^f, x^v, y)}{GDF_i^{SR}(x^f, x^v, y)}, \tag{21}$$

where $GDF_i^{SR}(x^f, x^v, y)$ is the optimal value of θ in model (17).

Note that $GPCU_i(x, x^f, y) \underset{<}{\overset{\geq}{=}} 1$ since $DF_i^{SR}(x^f, x^v, y) \underset{<}{\overset{\geq}{=}} GDF_i^{SR}(x^f, x^v, y)$. This lack of bound complicates its interpretation. If one prefers a bounded version, then we recommend to use the traditional input-oriented plant capacity measure (Definition 3.3). This graph input-oriented plant capacity utilization $GPCU_i$ compares the minimum amount of variable inputs for given amounts of outputs with the minimum amount of variable inputs with output levels where production is an mpss. It answers the question how the amount of variable inputs compatible with the mpss production must be scaled down or up to produce the current amount of outputs (see online supplementary material D for more details).

Proposition 4.3 *The following relations between the original graph as well as the special case graph output- and graph input-oriented plant capacity utilization notions as well as their components can be established:*

(i) $DF_i^{SR}(x^f, x^v, 0) \leq GDF_i^{SR}(x^f, x^v, y) \leq GDF_o^{SR}(x^f, x^v, y) \leq DF_o^f(x^f, y)$;
(ii) $PCU_o(x, x^f, y) \leq GPCU_o(x^v, x^f, y)$;
(iii) $GPCU_i(x^v, x^f, y) \leq PCU_i(x^v, x^f, y)$.
(iv) $GPCU(x, x^f, y) \underset{<}{\overset{\geq}{=}} GPCU_o(x, x^f, y) \underset{<}{\overset{\geq}{=}} GPCU_i(x, x^f, y)$
(v) $GPCU(x, x^f, y) \leq PCU_i(x^v, x^f, y)$
(vi) $GPCU(x, x^f, y) \underset{<}{\overset{\geq}{=}} PCU_o(x, x^f, y)$ *and* $GPCU(x, x^f, y)^{-1} \geq PCU_o(x, x^f, y)$.

Proof See online supplementary material C. □

Note that $GPCU(x, x^f, y)$ is in general different from its special input- and output-oriented graph versions $GPCU_i(x, x^f, y)$ and $GPCU_o(x, x^f, y)$, respectively. Furthermore, $GPCU(x, x^f, y)$ is also distinct from the traditional input- and output-oriented plant capacity concepts $PCU_i(x, x^f, y)$ and $PCU_o(x, x^f, y)$, respectively. Finally, $GPCU_i(x, x^f, y)$ and $GPCU_o(x, x^f, y)$ differ in general from $PCU_i(x, x^f, y)$ and $PCU_o(x, x^f, y)$, respectively.

4.2 Link to graph capacity measures based on directional distance functions

The directional distance function $E_D(x, y, g_x, g_y)$ proposed by Chambers et al. (1998) is defined by

$$E_D(x, y; g_x, g_y) = \max\{\beta \mid (x - \beta g_x, y + \beta g_y) \in T\}. \tag{22}$$

This distance function simultaneously seeks to expand outputs and contract inputs in the direction of the vector $(-g_x, g_y) \in \mathbb{R}_-^N \times \mathbb{R}_+^M$. The latter directional vector determines how the input-output vector (x, y) is projected onto the boundary of T at $(x - \beta^* g_x, y + \beta^* g_y)$, whereby $\beta^* = E_D(x, y; g_x, g_y)$.[8]

Partitioning the input vector x into fixed x^f and variable x^v, the sub-vector directional distance function is defined by

$$E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y) = \max\{\beta \mid (x^f, x^v - \beta g_{x^v}, y + \beta g_y) \in T\}. \tag{23}$$

Based on models (22) and (23), we define the directional plant capacity utilization $DPCU$ as follows:

[8] See Russell and Schworm (2011) for an almost complete overview of graph or non-oriented efficiency measures, including the directional distance function.

Definition 4.4 The directional plant capacity utilization $DPCU$ is defined as follows:

$$DPCU(x^f, x^v, y; g_{x^f}, g_x, g_y) = \frac{E_D(x, y; g_x, g_y)}{E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)}, \tag{24}$$

where $E_D(x, y; g_x, g_y)$ and $E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)$ are directional efficiency measures including respectively excluding the variable inputs as defined in (22) and (23).

Färe and Grosskopf (2000) are the first to use the directional distance function as a tool for defining a theoretical graph-oriented plant capacity utilization indicator. Färe and Grosskopf (2000) define the directional distance function when the variable inputs are unrestricted as follows:

$$E_D^{SR}(x^f, y; g_{x^f}, g_y) = \max\{\beta \mid (x^f - \beta g_{x^f}, x^v, y + \beta g_y) \in T, x^v \geq 0.\}. \tag{25}$$

In fact, they expand the outputs and contract the fixed inputs in the graph-oriented plant capacity measure using the directional distance function. This leads to the following definition of the directional plant capacity utilization $DPCU_{FG}$:

Definition 4.5 The directional plant capacity utilization $DPCU_{FG}$ is defined as follows:

$$DPCU_{FG}(x^f, x^v, y; g_{x^f}, g_x, g_y) = \frac{E_D(x, y; g_x, g_y)}{E_D^{SR}(x^f, y; g_{x^f}, g_y)}, \tag{26}$$

where $E_D(x, y; g_x, g_y)$ and $E_D^{SR}(x^f, y; g_{x^f}, g_y)$ are directional efficiency measures including and excluding the variable inputs as defined in (22) and (25) respectively.

Note that the definitions of the directional plant capacity utilization $DPCU$ and $DPCU_{FG}$ differ slightly from the original definition in Färe and Grosskopf (2000) in that no unity is added to both numerator and denominator. Note furthermore that the numerator of the directional plant capacity utilizations concepts (24) and (26) is the same and the only difference between these is in the denominator, where our proposed method uses (23) and the Färe and Grosskopf (2000) method applies (25).

We now show in the following propositions that the building blocks needed for computing all plant capacity measures hitherto defined in this contribution can be obtained from the directional distance functions $E_D(x, y; g_x, g_y)$ and $E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)$ in (24) by choosing appropriate direction vectors. For the proofs, we refer to online supplementary material C.

Proposition 4.4 *Assume that (θ^*, φ^*) is an optimal solution obtained by solving the following model:*

$$DF_o^f(x^f, y) = \max\left\{\varphi \mid \theta \geq 0, \varphi \geq 0, (x^f, \theta x^v, \varphi y) \in T\right\}. \tag{27}$$

Letting

$$g_{x^v} = \frac{(1-\theta^*)}{DF_o^f(x^f, y)} x^v, \quad and$$
$$g_y = \frac{(\varphi^* - 1)}{DF_o^f(x^f, y)} y,$$

then $DF_o^f(x^f, y) = E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)$.
Note that if $(\theta^*, \varphi^*) = (1, 1)$, then we put $E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y) = 1$.

Proposition 4.5 *Assume that (θ^*, φ^*) is an optimal solution obtained by solving the following model:*

$$DF_o(x, y) = \max\left\{\varphi \mid \theta \leq 1, \varphi \geq 1, (x^f, \theta x^v, \varphi y) \in T\right\}. \tag{28}$$

Letting

$$g_{x^v} = \frac{(1-\theta^*)}{DF_o(x, y)} x^v, \quad \text{and}$$
$$g_y = \frac{(\varphi^*-1)}{DF_o(x, y)} y,$$

then $DF_o(x, y) = E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)$.
Note that if $(\theta^*, \varphi^*) = (1, 1)$, then we put $E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y) = 1$.

Proposition 4.6 *Assume that (θ^*, φ^*) is an optimal solution obtained by solving the following model:*

$$DF_i^{SR}(x^f, x^v, 0) = \min\{\theta \mid \theta \geq 0, \varphi \geq 0, (x^f, \theta x^v, \varphi y) \in T\}. \tag{29}$$

Letting

$$g_{x^v} = \frac{(1-\theta^*)}{DF_i^{SR}(x^f, x^v, 0)} x^v, \quad \text{and}$$
$$g_y = \frac{(\varphi^*-1)}{DF_i^{SR}(x^f, x^v, 0)} y,$$

then $DF_i^{SR}(x^f, x^v, 0) = E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)$.
Note that if $(\theta^*, \varphi^*) = (1, 1)$, then we put $E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y) = 1$.

Proposition 4.7 *Assume that (θ^*, φ^*) is an optimal solution obtained by solving the following model:*

$$DF_i^{SR}(x^f, x^v, y) = \min\{\theta \mid \theta \leq 1, \varphi \geq 1, (x^f, \theta x^v, \varphi y) \in T\}. \tag{30}$$

Letting

$$g_{x^v} = \frac{(1-\theta^*)}{DF_i^{SR}(x^f, x^v, y)} x^v, \quad \text{and}$$
$$g_y = \frac{(\varphi^*-1)}{DF_i^{SR}(x^f, x^v, y)} y,$$

then $DF_i^{SR}(x^f, x^v, y) = E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)$.
Note that if $(\theta^*, \varphi^*) = (1, 1)$, then we put $E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y) = 1$.

Proposition 4.8 *Assume that (θ^*, φ^*) is an optimal solution obtained by solving the following model:*

$$E_G^f(x^f, x^v, y) = \max\{\frac{\varphi}{\theta} \mid \theta \leq 1, \varphi \geq 1, (x^f, \theta x^v, \varphi y) \in T\}. \tag{31}$$

Letting

$$g_{x^v} = \frac{(1-\theta^*)}{E_G^f(x^f, x^v, y)} x^v, \quad \text{and}$$

$$g_y = \frac{(\varphi^* - 1)}{E_G^f(x^f, x^v, y)} y,$$

then $E_G^f(x^f, x^v, y) = E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)$.
Note that if $(\theta^*, \varphi^*) = (1, 1)$*, then we put* $E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y) = 1$.

Proposition 4.9 *Assume that* (θ^*, φ^*) *is an optimal solution obtained by solving the following model:*

$$E_G^{SR}(x^f, x^v, y) = \max\{\frac{\varphi}{\theta} \mid \theta \geq 0, \varphi \geq 0, (x^f, \theta x^v, \varphi y) \in T\}. \quad (32)$$

Letting

$$g_{x^v} = \frac{(1 - \theta^*)}{E_G^{SR}(x^f, x^v, y)} x^v, \quad \textit{and}$$
$$g_y = \frac{(\varphi^* - 1)}{E_G^{SR}(x^f, x^v, y)} y,$$

then $E_G^{SR}(x^f, x^v, y) = E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y)$.
Note that if $(\theta^*, \varphi^*) = (1, 1)$*, then we put* $E_D^{SR}(x^v, x^f, y; g_{x^v}, g_y) = 1$.

In Propositions 4.5, 4.7 and 4.8, g_{x^v} and g_y are non-negative: therefore, the variable inputs and outputs adjust in the direction of $(-g_{x^v}, g_y) \in \mathbb{R}_-^{N_v} \times \mathbb{R}_+^M$. In Proposition 4.6, g_{x^v} is non-negative and g_y is non-positive: i.e., the direction vector is $(-g_{x^v}, g_y) \in \mathbb{R}_-^{N_v} \times \mathbb{R}_-^M$. In Proposition 4.4, g_y is non-negative, but g_{x^v} can sometimes be negative: hence, in this case the direction vector is $(-g_{x^v}, g_y) \in \mathbb{R}^{N_v} \times \mathbb{R}_+^M$. In Proposition 4.9, g_{x^v} and g_y can sometimes be positive or negative: therefore, in this case the direction vector is $(-g_{x^v}, g_y) \in \mathbb{R}^{N_v} \times \mathbb{R}^M$. Therefore, one may consider the graph-oriented plant capacity utilization indicator (24) based on the directional distance functions (22) and (23) as a special case of the traditional output- and input-oriented plant capacity measures as well as our new graph-oriented plant capacity utilization index (18).

However, we are rather critical about the graph-oriented plant capacity utilization indicator (26) based on the directional distance functions proposed by Färe and Grosskopf (2000) [i.e., (25)]. Our proposed method (23) has two main differences with the Färe and Grosskopf (2000) method (25). First, in our method we can adjust (contract and expand) the outputs as well as the variable inputs, while the Färe and Grosskopf (2000) method expands the outputs and contracts the fixed inputs. However, adjusting fixed inputs is in principle impossible (unless one adopts an alternative, more flexible definition of input fixity), while variable inputs are under the control of management.[9] Hence, it seems that adjusting variable inputs is more reasonable than contracting fixed inputs. Therefore, the obtained targets from our method are more acceptable for decision makers.

Second, in model (25) proposed by Färe and Grosskopf (2000) both fixed and variable inputs can adjust, while our proposed model (23) adjusts only the variable inputs. In fact, in the Färe and Grosskopf (2000) method (25), x^v is a decision variable such that it is non-negative ($x^v \geq 0$). Therefore, at optimality, it can be increased or decreased from its existing level, and the fixed inputs adjust in the direction of g_{x^f}. But, our model only adjusts the variable inputs in the direction of g_{x^v} (depending on the precise sign of this direction vector)

[9] Note that Yang and Fukuyama (2018) offer an output-oriented plant capacity notion with a directional distance function, but do not consider reductions in fixed inputs.

while the fixed inputs do not change. Note that in the Färe and Grosskopf (2000) model (25), one cannot select a suitable direction to reach the target of the input-oriented efficiency measure $DF_i^{SR}(x^f, x^v, 0)$, while our model can cover this input-oriented efficiency measure as well: therefore, it is more general.

5 Numerical example

We illustrate the ease of implementing these new graph plant capacity definitions by using a small example of artificial data. We refer to online supplementary material B for an overview on how to compute the necessary components of the plant capacity notions assuming a convex non-parametric technology under VRS. Table 1 contains 16 fictitious observations with two inputs generating a single output: one input is variable, the other one is fixed.

The results of the traditional input- and output plant capacity measures and their components are reported in Table 2. Observe that the observations with a unit value for $PCU_i(.)$ and $PCU_o(.)$ are different in general: solely observations 4, 5 and 7 have a unity plant capacity in both cases. This confirms that both these plant capacity measures evaluate different things.

The results of the new graph plant capacity concept $GPCU(x, x^f, y)$ as well as its special input- and output-oriented graph versions $GPCU_i(x, x^f, y)$ and $GPCU_o(x, x^f, y)$ are reported in Table 3. This same table also contains their respective component efficiency measures. Observe that observations 1, 5 and 14 with a unit value for $GPCU(x, x^f, y)$ have different values for $GPCU_i(x, x^f, y)$ and $GPCU_o(x, x^f, y)$. By contrast, observations 4, 6-7, 9 and 11 are unity for all three graph capacity notions. For the observations which are different from unity, all three graph capacity notions in general differ.

Figure 1 shows the (x^v, y)-combinations determined by the optimal solutions of those models needed for computing $PCUi(.)$, $PCUo(.)$ and $GPCU(.)$ applied to observation 12 (black solid circle). The values mentioned below can be found in Tables 2 and 3 except

Table 1 Numerical example of 16 fictitious observations

Observations	x^v	x^f	y
1	7	6	2
2	3	5	2
3	5	4	3
4	6	3	3
5	7	4	3
6	4	9	4
7	11	3	4
8	5	6	4
9	6	3	4
10	6	7	5
11	5	8	5
12	8	6	5
13	10	5	5
14	6	10	6
15	7	8	6
16	10	7	6

Table 2 Input- and output-oriented plant capacity utilization and components

	$PCU_i(.)$			$PCU_o(.)$		
	$DF_i^{SR}(x^f, x^v, y)$	$DF_i^{SR}(x^f, x^v, 0)$	$PCU_i(.)$	$DF_o(.)$	$DF_o^f(.)$	$PCU_o(.)$
1	0.4286	0.4286	1.0000	2.6250	2.7500	0.9545
2	1.0000	1.0000	1.0000	1.0000	2.5000	0.4000
3	0.9000	0.9000	1.0000	1.1538	1.5000	0.7692
4	1.0000	1.0000	1.0000	1.3333	1.3333	1.0000
5	0.6429	0.6429	1.0000	1.5000	1.5000	1.0000
6	1.0000	0.7500	1.3333	1.0000	1.5000	0.6667
7	0.5455	0.5455	1.0000	1.0000	1.0000	1.0000
8	0.9500	0.6000	1.5833	1.0577	1.3750	0.7692
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0.9333	0.5000	1.8667	1.0400	1.2000	0.8667
11	1.0000	0.6000	1.6667	1.0000	1.2000	0.8333
12	0.7750	0.3750	2.0667	1.0750	1.1000	0.9773
13	0.8000	0.3000	2.6667	1.0000	1.0000	1.0000
14	1.0000	0.5000	2.0000	1.0000	1.0000	1.0000
15	1.0000	0.4286	2.3333	1.0000	1.0000	1.0000
16	1.0000	0.3000	3.3333	1.0000	1.0000	1.0000

Table 3 Graph plant capacity utilization, graph input- and output-oriented plant capacity utilization, and components

	$GPCU(.)$			$GPCU_i(.)$		$GPCU_o(.)$	
	$E_G^f(.)$	$E_G^{SR}(.)$	$GPCU(.)$	$GDF_i^{SR}(.) = \bar{\lambda}$	$GPCU_i(.)$	$GDF_o^{SR}(.)$	$GPCU_o(.) = APCU(., \bar{\lambda})$
1	2.9815	2.9815	1.0000	0.7714	0.5556	2.3000	1.1413
2	1.0000	1.1786	0.8485	1.8667	0.5357	2.2000	0.4545
3	1.1538	1.2069	0.9560	1.1600	0.7759	1.4000	0.8242
4	1.3333	1.3333	1.0000	1.0000	1.0000	1.3333	1.0000
5	1.6897	1.6897	1.0000	0.8286	0.7759	1.4000	1.0714
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	1.8333	1.8333	1.0000	0.5455	1.0000	1.0000	1.0000
8	1.0577	1.0648	0.9933	1.0800	0.8796	1.1500	0.9197
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0714	1.1077	0.9673	0.8667	1.0769	0.9600	1.0833
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.2903	1.3630	0.9467	0.6750	1.1481	0.9200	1.1685
13	1.2500	1.5714	0.7955	0.5600	1.4286	0.8800	1.1364
14	1.0000	1.0000	1.0000	0.6667	1.5000	0.6667	1.5000
15	1.0000	1.1667	0.8571	0.7143	1.4000	0.8333	1.2000
16	1.0000	1.5385	0.6500	0.5200	1.9231	0.8000	1.2500

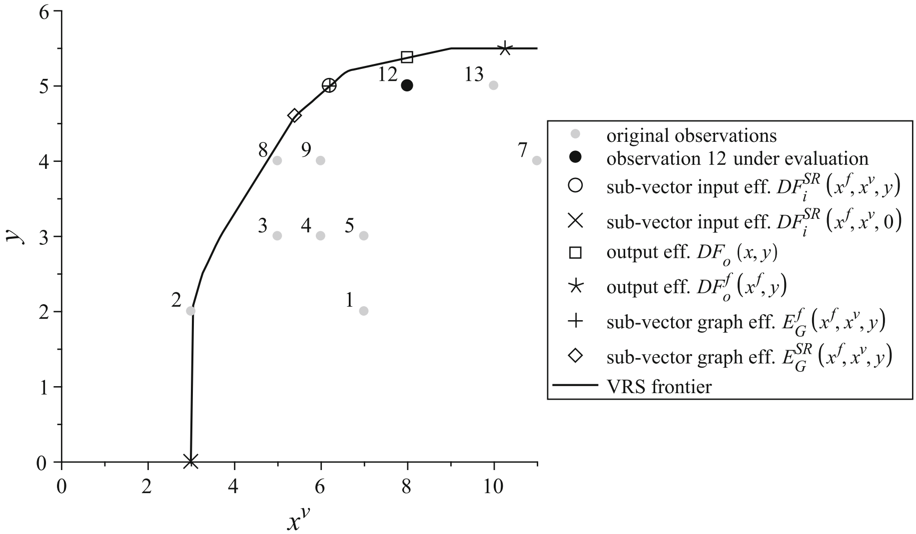

Fig. 1 Visualization of the different components of PCU_i, PCU_o and $GPCU$ applied to observation 12

for the values of φ^* and θ^*. Observe in Fig. 1 the original observations (gray solid circles with labels referring to the observation numbering) with fixed input smaller than or equal to the fixed input of observation 12 and the corresponding frontier. Applying (11) to compute $DF_i^{SR}(x^f, x^v, y)$ on this observation results in the efficiency value 0.7750 with $\varphi^* = 1$ and $\theta^* = 0.7750$. Consequently, the variable input is reduced to 6.2 without expanding the output. The resulting (x^v, y)-combination is depicted by the black circle (○) located on the frontier. Applying (9) to compute $DF_i^{SR}(x^f, x^v, 0)$ yields 0.3750 with $\varphi^* = 0$ and $\theta^* = 0.3750$. So, by allowing to reduce outputs to the level of zero, the variable input can be reduced further from 6.2 to 3. Combining the optimal variable input and output level realizes the point visualized by a diagonal cross (×). Since there is only one variable input and one output in this example, $PCU_i(x, x^f, y)$ boils down to the ratio of the horizontal distance from the point ○ to the y-axis over the horizontal distance from the point × to the y-axis.

Using (10) to compute $DF_o(x, y)$ for observation 12 leads to the optimal value 1.0750 with $\varphi^* = 1.0750$ and $\theta^* = 1$. Now, the output is maximally increased without reducing the variable input. The corresponding optimal (x^v, y)-combination is visualized by the black box (□) in Fig. 1. Using (8) to determine $DF_o^f(x^f, y)$ results in the optimal value 1.1000 with $\varphi^* = 1.1000$ and $\theta^* = 1.2813$. So, by allowing an increase of the variable input, the output can be increased by this factor 1.1000. The corresponding optimal (x^v, y)-combination is visualized by the asterisk (⋆) at (10.25, 5.5). Since there is only one variable input and one output in this example, $PCU_o(x, x^f, y)$ corresponds with the ratio of the vertical distance from the point □ to the x^v-axis over the vertical distance from the point ⋆ to the x^v-axis.

Computing the sub-vector graph efficiency $E_G^f(x^f, x^v, y)$ defined by (16) for observation 12 leads to the optimal value 1.2903 with $\varphi^* = 1$ and $\theta^* = 0.7750$. Obviously, the maximal ratio $\frac{\varphi^*}{\theta^*}$ is realized by reducing the variable input and keeping the output at the same level. This results in the optimal (x^v, y)-combination depicted in Fig. 1 by the cross (+). Note that this point coincides for observation 12 with the optimal point realized by $DF_i^{SR}(x^f, x^v, y)$ and visualized by ○. Using (17), the sub-vector graph efficiency $E_G^{SR}(x^f, x^v, y)$ for observation yields the optimal ratio $\frac{\varphi^*}{\theta^*} = 1.3630$ with $\varphi^* = 0.9200$ and $\theta^* = 0.6750$. Thus, the optimal ratio is obtained by reducing both the variable input and the output. The corresponding optimal (x^v, y)-combination is visualized by the diamond (◇) at (5.4, 4.6). The ratio of the ratios $\frac{\varphi^*}{\theta^*}$ mentioned above yields $GPCU(x, x^f, y) = 0.9467$.

We explore the differences between these graph plant capacity notions in more detail in the empirical illustration in the next Sect. 6.

6 Empirical illustration

6.1 Secondary data set

For an overview of the models used in this empirical illustration assuming a convex non-parametric technology under VRS, we refer to online supplementary material B. For the sake of replication by the reader, we select a secondary data set from the *Journal of Applied Econometrics* Data Archive[10] and opt for an unbalanced panel of three years of French fruit producers from Ivaldi et al. (1996) based on annual accounting data collected in a survey. These farms were selected on mainly two criteria: (i) the apple production must be positive, and (ii) the orchard acreage is five acres at least. This short panel covers three successive years from 1984 to 1986. The technology consists of three aggregate inputs producing two aggregate outputs. The three inputs are (i) capital (including land), (ii) labor, and (iii) materials; while the two outputs are (i) the apple production, and (ii) an aggregate of other products. Descriptive statistics for the 405 observations in total and details on the definitions of all variables are available in Appendix 2 of Ivaldi et al. (1996). Note that the short length of the panel (just three years) warrants the use of an intertemporal technology that essentially ignores any eventual technical change.

6.2 Empirical results

The descriptive statistics for the traditional input- and output-oriented plant capacity utilization measures as well as their components are reported in Table 4. We report the average, the standard deviation, and the minima and maxima depending on the context. These descriptive statistics seem to confirm that both concepts clearly differ from one another (see Cesaroni et al. 2017 for a detailed empirical analysis confirming this observation).

The descriptive statistics for new graph plant capacity concept $GPCU(x, x^f, y)$ as well as its special input- and output-oriented graph versions $GPCU_i(x, x^f, y)$ and $GPCU_o(x, x^f, y)$ are reported in Table 5. Also the component efficiency measures are listed in the same table. We can make the following observations. First, the descriptive statistics indicate that all three graph capacity notions in general seem to differ. Furthermore, comparing the minimum and maximum of $GPCU_i(x, x^f, y)$ shows that it lacks a natural bound. Making the same comparison for $GPCU_o(x, x^f, y)$ equally reveals the lack of a natural bound.

Second, note that based on Proposition 4.2, $GPCU_o(x, x^f, y) = APCU_o(x, x^f, y, \bar{\lambda})$ and $GDF_o(x, x^f, y) = ADF_o^f(x, x^f, y, \bar{\lambda})$. Hence, the two last columns of Table 5 report the biased and unbiased attainable output-oriented plant capacity utilisation at level $\bar{\lambda} = \theta^*$, respectively, where θ^* is the optimal value of θ in model (17). Hence, $\bar{\lambda} = GDF_i^{SR}(x, x^f, y)$

[10] See the web site: http://qed.econ.queensu.ca/jae/.

Table 4 Descriptive statistics of input- and output-oriented plant capacity utilization

	$PCU_i(.)$			$PCU_o(.)$		
	$DF_i^{SR}(x^f, x^v, y)$	$DF_i^{SR}(x^f, x^v, 0)$	$PCU_i(.)$	$DF_o(.)$	$DF_o^f(.)$	$PCU_o(.)$
Average	0.5881	0.4233	1.7337	3.4924	5.4149	0.7105
SD	0.1924	0.1950	1.6360	2.6312	4.6781	0.2211
Minimum	0.1868	0.0473	1	1	1	0.0701
Maximum	1	1	21.1414	16.2869	35.2953	1

which is reported in the fifth column of Table 5. While the attainable output-oriented plant capacity utilization $APCU_o(x, x^f, y, \bar{\lambda})$ depends on the attainable level $\bar{\lambda}$ determined by the decision maker, the defined attainability in Definition 4.2 (i.e., $GPCU_o(x, x^f, y) = APCU_o(x, x^f, y, \bar{\lambda})$) can be a good choice to determine some reasonable bound on the available variable inputs. As can be seen in the fifth column of Table 5, the average of attainability level $\bar{\lambda}$ is 1.0463 with the standard deviation 0.4366. With this proposed attainability level $\bar{\lambda}$ we obtain on average a 4.2375 output magnification, while the maximum increase in outputs amounts to 33.1387 times. It suffices to put this in contrast with the biased plant capacity measure $DF_o^f(x, x^f, y)$ in Table 4 (column 6). There is more variation in $DF_o^f(x, x^f, y)$ as indicated by the standard deviation and the range is even bigger: the maximum increase in outputs amounts to 35.2953 times. Our approach clearly suffers less from such extreme magnifications and therefore remains somewhat more attainable based on this output magnification.

From the viewpoint of input magnification, Kerstens et al. (2019b) define the following critical point U_p as follows:

$$U_p = DF_i^{SR}(x_p^f, x_p^v, DF_o^f(x_p^f, y_p)y_p), \tag{33}$$

where $DF_o^f(x_p^f, y_p)$ and $DF_i^{SR}(x_p^f, x_p^v, y_p)$ are defined before in (3) and (4), respectively. It can be interpreted as the minimal expansion of variable inputs needed to produce the maximum plant capacity outputs. By solving model (33), the average of critical point U_p is 2.597 with a standard deviation of 1.621. Also, its minimum and maximum are 0.513 and 8.544, respectively. Based on the average of this critical point U_p one needs about 2.60 times more variable inputs than currently in use to reach maximum plant capacity outputs, while one can needs magnifying variable inputs by only a factor 1.0463 in Definition 4.2 (i.e., $GPCU_o(x, x^f, y) = APCU_o(x, x^f, y, \bar{\lambda})$). Note that the variation in this factor U_p is rather substantial. The maximal magnification factor of 8.544 is clearly implausible in reality. These very strong requirements on the availability of variable inputs clearly cast doubts on the plausibility of the traditional output-oriented plant capacity measure (see Kerstens et al. 2019b for more details).

To test whether any of the above results are statistically significant, we choose the formal test statistic proposed by Li (1996) and refined by Fan and Ullah (1999) and Li et al. (2009) (henceforth Li-test). This Li-test has the null hypothesis that both distributions are equal for a given efficiency score or plant capacity notion. Its alternative hypothesis is simply that both distributions differ. This test is valid for both dependent and independent variables: dependency is a characteristic of frontier estimators, since frontier efficiency depend on sample size, among others. Table 6 reports the Li-test statistics for all plant capacity concepts discussed in this contribution. A glance at Table 6 shows that all plant capacity concepts

Table 5 Descriptive statistics of various graph plant capacity utilization concepts

	$GPCU(.)$			$GPCU_i(.)$		$GPCU_o(.)$	
	$E_G^f(.)$	$E_G^{SR}(.)$	$GPCU(.)$	$\bar{\lambda} = GDF_i^{SR}(.)$	$GPCU_i(.)$	$GDF_o^{SR}(.) = ADF_o^f(., \bar{\lambda})$	$GPCU_o(.) = APCU_o(., \bar{\lambda})$
Average	3.6971	3.9243	0.9430	1.0463	0.6742	4.2375	1.0240
SD	2.7219	2.8763	0.1049	0.4366	0.5713	4.1750	0.4875
Minimum	1	1	0.1521	0.1333	0.1842	0.2804	0.0835
Maximum	16.2869	16.4006	1	2.9549	7.5025	33.1387	3.5658

Table 6 Li-test among all unbiased plant capacity notions

	$PCU_i(.)$	$PCU_o(.)$	$GPCU(.)$	$GPCU_i(.)$	$GPCU_o(.)$
$PCU_i(.)$		74.606***	143.6798***	124.5037***	17.558***
$PCU_o(.)$	74.606***		71.1327***	21.742***	20.6112***
$GPCU(.)$	143.6798***	71.1327***		145.117***	110.6579***
$GPCU_i(.)$	124.5037***	21.742***	145.117***		55.832***
$GPCU_o(.)$	17.558***	20.6112***	110.6579***	55.832***	

Li test: critical values at 1% level = 2.33(***); 5% level = 1.64(**); 10%level = 1.28(*)

follow two by two significantly different distributions and thus capture an independent part of reality.

7 Conclusions

While the output-oriented plant capacity concept has been around for about three decades and has been the basis for quite some empirical applications, the input-oriented plant capacity concept is of more recent date. However, this original output-oriented plant capacity utilization suffers from the so-called attainability issue, which has led Kerstens et al. (2019b) to define an attainable output-oriented plant capacity notion. This contribution has taken a next logical step by looking for graph efficiency measures to define some new graph-oriented plant capacity concepts. Apart from some new definitions, relations between these capacity concepts have been established. A small numerical example has illustrated these various concepts and an empirical application on a secondary data set has revealed the factual differences between these different notions.

We end by outlining some potential avenues for future research. First, this contribution has focused on what has been called short-run plant capacity concepts (see Cesaroni et al. 2019). Recently, Cesaroni et al. (2019) also define long-run output- and input-oriented plant capacity concepts. It remains to be seen whether our analysis can be extended to these long-run plant capacity concepts as well. Second, convexity has been shown to have an impact on input-and output-oriented plant capacity measures (see Cesaroni et al. 2017). Furthermore, convexity turns out to affect long-run plant capacity concepts and alternative cost-based capacity concepts as well (e.g., Kerstens et al. 2019a). Therefore, it is certainly useful to assess the impact of convexity on these new graph-oriented plant capacity measures. Third, there are some indications that slacks may play a role in the measurement of plant capacity utilization (e.g., Dupont et al. 2002, or Vestergaard et al. 2003). Of course, on a priori grounds one would expect that slacks play less a role for graph-oriented plant capacity that are more likely to project on the efficient subset of technology than for output- or input-oriented plant capacity notions that are more likely to project on the isoquant. But, this conjecture certainly merits further attention. Fourth, the current lack of bounds for the input- and output-oriented versions of our graph plant capacity measure deserves to be remedied. Perhaps the use of alternative graph or non-oriented efficiency measures (see Russell and Schworm 2011) may provide a way out.

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